MATH 1010 Tutorial Dec. 3rd (English III)

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Schedule:

5:30 – 6:05 Tutorial presentation
(5 Questions will be discussed)

6:05 – 6:15 Q & A
(Hosted by: YI, Tianhan)

Remark:

- You can visit yzwang.xyz or Blackboard to download the tutorial notes.

- MathGym (Faculty TA Q&A Centre) is now open. Please visit mathgym.math.cuhk.edu.hk for more information.

1. Evaluate the integrals:

(a) $\int_{\frac{\pi}{5}}^{\pi} |\cos x|\, dx$

$= \int_{\frac{\pi}{5}}^{\frac{\pi}{2}} \cos x\, dx + \int_{\frac{\pi}{2}}^{\pi} (-\cos x)\, dx$

$= \sin(x)\Big|_{x=\frac{\pi}{5}}^{\frac{\pi}{2}} - \sin(x)\Big|_{x=\frac{\pi}{2}}^{\pi}$

$= (1 - \sin\frac{\pi}{5}) - (0 - 1)$

$= 2 - \sin\frac{\pi}{5}$

(b) $\int_{-3}^{7} (x - 6|x|)\, dx$

$= \int_{-3}^{0} x + 6x\, dx + \int_{0}^{7} x - 6x\, dx$

$= \int_{-3}^{0} 7x\, dx + \int_{0}^{7} -5x\, dx$

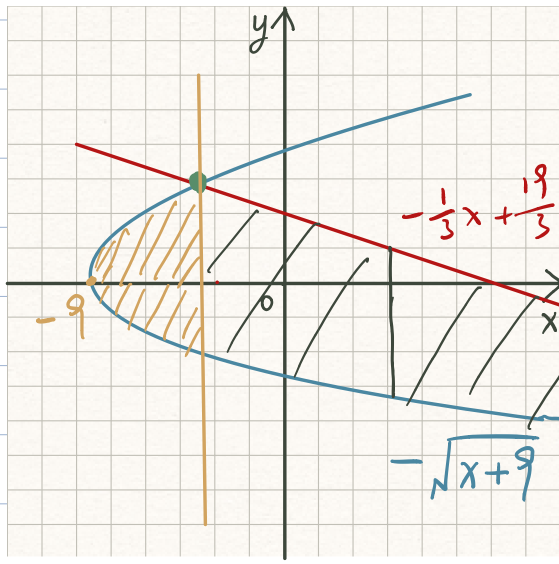
$= (\frac{7}{2}x^2)\Big|_{x=-3}^{0} - (\frac{5}{2}x^2)\Big|_{x=0}^{7}$

$= \frac{7}{2}(0-9) - \frac{5}{2}(49-0)$

$= \frac{1}{2}(-7\times 9 - 5\times 49)$ 7×35

$= \frac{1}{2}(-7\times 44) = -154$

2. Find the area between the curves $x+3y=19$ and $x+9=y^2$.



$y=-\frac{1}{3}x+\frac{19}{3}$

or $x=19-3y$

$x=y^2-9$

$y^2-9+3y=19$

$y^2+3y-28=0$

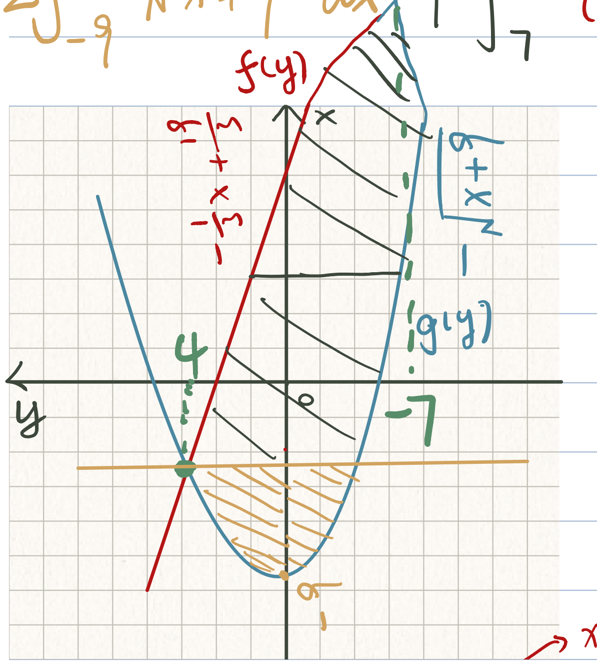
$(y+7)(y-4)=0$

Hence $y=-7$ or 4

and $x=40$ or 7

Hence the intersection points are $(40,-7)$ $(7,4)$

$$2\int_{-9}^{7}\sqrt{x+9}\,dx+\int_{7}^{40}\left(-\frac{1}{3}x+\frac{19}{3}\right)-\left(-\sqrt{x+9}\right)dx=\frac{1331}{6}$$



$$\int_{-7}^{4}f(y)-g(y)\,dy$$

$$=\int_{-7}^{4}19-3y-(y^2-9)\,dy$$

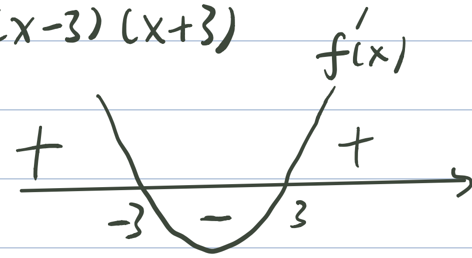
$$=\frac{1331}{6}$$

3. Let $f(x) = \int_0^x \frac{t^2 - 9}{1 + \cos^2(t)} \, dt.$ $\rightarrow f(x) = (x^2-9) \int_0^x \frac{1}{1+\cos^2(t)} dt$

Find the value of x where the local maximum of $f(x)$ occur.

By the Fundamental Theorem of Calculus (FTC)

$$f'(x) = \frac{x^2-9}{1+\cos^2(x)} = \frac{1}{1+\cos^2(x)} \cdot (x-3)(x+3)$$



Hence local maximum attains at -3.

4. Let $g(x) = \int_3^{x^2} e^{-t^3} dt$.

Find $g'(x)$.

Let $F(x) = \int_3^{x} e^{-t^3} dt$.

Then $g(x) = F(x^2)$.

Hence $g'(x) = \frac{d}{dx} x^2 \cdot F'(x^2)$ (by chain rule)

$= 2x \cdot e^{-x^6}$ (by FTC)

5. Find $\frac{d}{dx}\left(\int_{-2}^{\sqrt{x}} \frac{\cos t}{t^2}\, dt\right)$

Let $F(x) = \int_{-2}^{x} \frac{\cos t}{t^2}\, dt$

Let $g(x) = F(\sqrt{x})$.

Then $g'(x) = \frac{d}{dx}\sqrt{x} \cdot F'(\sqrt{x})$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}} \cdot \frac{\cos(\sqrt{x})}{(\sqrt{x})^2}$$

$$= \frac{1}{2\sqrt{x}} \cdot \frac{\cos(\sqrt{x})}{x}$$

$$= \frac{\cos(\sqrt{x})}{2\sqrt{x}\, x}$$

Q: $\frac{d}{dx}\left(\int_0^x e^{xt}\,dt\right) = ?$

Let $y = xt$, then $t = \frac{y}{x}$.

Hence $\int_0^x e^{xt}\,dt = \int_0^{x^2} e^y \, d\left(\frac{y}{x}\right)$

$= \frac{1}{x}\int_0^{x^2} e^y \, dy$