MATH 1010 Tutorial Oct. 28th (English III)

Presenter: WANG, Yizi
Email: yzwang@math.cuhk.edu.hk

Schedule:

~~5:45~~ 5:52 – 6:05 Tutorial presentation
(5 Questions will be discussed)

6:05 – 6:15 Q & A
(Hosted by: YI, Tianhan)

Remark:

- You can visit yzwang.xyz or Blackboard to download the tutorial notes.

- MathGym (Faculty TA Q&A Centre) is now open. Please visit mathgym.math.cuhk.edu.hk for more information.

Q1. Let $f(x) = x^2 - 5x + 6$.

(1) The critical point c of f is ?

(2) Determine the monotomicity of f on $(-\infty, c)$ and $(c, +\infty)$

(1) $f'(x) = 2x - 5$

Let $f'(x) = 0$.

Then $x = \frac{5}{2}$ is the critical point.

Hence $c = \frac{5}{2}$

(2) For $x < \frac{5}{2}$, $f'(x) < 0$.

For $x > \frac{5}{2}$, $f'(x) > 0$.

Hence f is increasing on $(\frac{5}{2}, +\infty)$

and f is decreasing on $(-\infty, \frac{5}{2})$

Q2. Let $f(x) = x - \ln(2x)$, $x > 0$

$= \ln(2) + \ln(x)$

(a) Critical point of f = ?

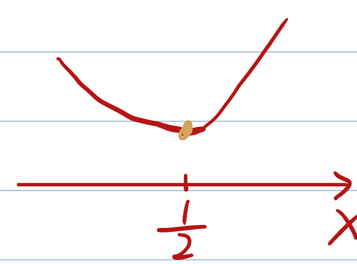
(b) Is f a maximum or minimum at the critical point? (Apply the First Derivative Test)

(a) $f'(x) = 1 - \frac{1}{x}$.

Let $f'(x) = 0$.
Then $x = 1$.

Hence f has critical point $x = \frac{1}{2}$.

(b) If $x < 1$, then $f'(x) < 0$
If $x > 1$, then $f'(x) > 0$



Hence $f(1)$ is the minimum of f.

Q3. Let $f(x) = e^x + e^{-x}$ on $[-1, \infty)$.

Then f is increasing on $\underline{(0, +\infty)}$

f is decreasing on $\underline{(-1, 0)}$

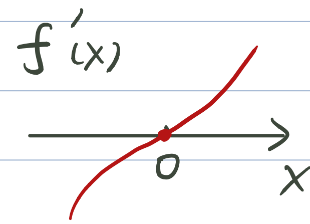
$f'(x) = e^x - e^{-x}$

Let $f'(x) = 0$, then we have $x = 0$.

$e^x = e^{-x}$

$e^{2x} = 1$

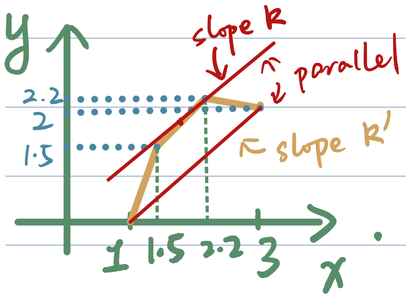
$x = 0$



$f''(x) = e^x + e^{-x} > 0$

Hence we have that $f'(x) > 0$ on $(0, +\infty)$, $f'(x) < 0$ on $(-1, 0)$.

Q4. Consider a function f with graph



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(a) Can we apply Mean Value Thm to f on $[1,3]$?

(b) Can we find $c \in (1,3)$ such that $f'(c) = \dfrac{f(3)-f(1)}{3-1}$?

(a) No. Since f is not differentiable at 1.5 and 2.2.

(b) Yes. $k = \dfrac{2.2-1.5}{2.2-1.5} = 1$

$k' = \dfrac{f(3)-f(1)}{3-1} = \dfrac{2-0}{3-1} = 1$

Q5. Let $f(x) = e^x$ and $g(x) = x+1$. Apply Racetrack Principle to show that $f(x) \geq g(x)$.

If $f(0) = g(0)$ and $f'(x) \geq g'(x)$ for $x > 0$, then $f(x) \geq g(x)$ for $x \geq 0$.

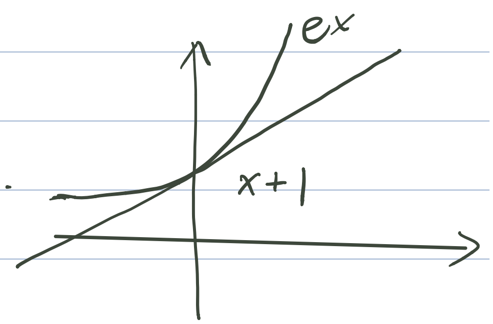
If $f(0) = g(0)$ and $f'(x) \leq g'(x)$ for $x < 0$, then $f(x) \geq g(x)$ for $x \leq 0$.

$f'(x) = e^x \qquad g'(x) = 1.$

Hence $f'(x) \geq g'(x)$ for $x > 0$.

$f(0) = 1 = g(0)$

Hence $f(x) \geq g(x)$ for $x \geq 0$.



For $x<0$, $f'(x) \leq g'(x)$.

Hence $f(x) \geq g(x)$ for $x \leq 0$.

Therefore, $f(x) \geq g(x)$ for $x \in \mathbb{R}$.