MATH 1010 Tutorial Oct. 22nd (English III)

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Schedule:

5:30 – 6:05 Tutorial presentation
(5 Questions will be discussed)

6:05 – 6:15 Q & A
(Hosted by: YI, Tianhan)

Remark:

- You can visit yzwang.xyz or Blackboard to download the tutorial notes.

- Math Gym (Faculty TA Q&A Centre) is now open. Please visit mathgym.math.cuhk.edu.hk for more information.

Q1. Find the derivative of

$$f(x) = e^{e^{x^2}}$$

Let $g(x) = e^{x^2}$

Then $g'(x) = 2x\, e^{x^2}$

Hence $f'(x) = g'(x)\, e^{g(x)}$

$= 2x\, e^{x^2} e^{e^{x^2}}$

Q2. Let $f(x) = \ln\left(\sqrt{\frac{(x-2)^{18}}{(x-1)^{16}}}\right)$

(a) Write out $f(x)$ using sums and/or differences of logarithmic expressions which do not contain the logarithms of products, quotients, or powers.

(b) Using (a) to find $f'(x)$.

(a) Note that $\sqrt{x^2} = |x|$.

Hence $\sqrt{\frac{(x-2)^{18}}{(x-1)^{16}}} = \frac{|x-2|^9}{|x-1|^8}$

Then $f(x) = \ln\left(\frac{|x-2|^9}{|x-1|^8}\right)$ $\quad \ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$$= \ln(|x-2|^9) - \ln(|x-1|^8)$$
$$= 9\ln(|x-2|) - 8\ln(|x-1|)$$

$\ln(a^b) = b\ln a$

(b) $\frac{d}{dx}|x| = \begin{cases} 1 & \text{if } x>0 \\ -1 & \text{if } x<0 \end{cases}$ i.e. $\frac{d}{dx}|x| = \frac{|x|}{x}$ $(x \neq 0)$

$$\frac{d}{dx}\ln|x| = \frac{1}{|x|}\cdot\frac{|x|}{x} = \frac{1}{x}$$

$$f'(x) = \frac{9}{x-2} - \frac{8}{x-1}$$

Q3. Compute $f'(x)$, $f''(x)$, $f'''(x)$, and then state a formula for $f^{(n)}(x)$, for

$$f(x) = \frac{1}{x}.$$

$f(x) = x^{-1}$ $\qquad \frac{d}{dx} x^n = n x^{n-1}$

$f'(x) = (-1)\, x^{-2}$

$f''(x) = (-1)(-2)\, x^{-3} = 2x^{-3}$ $\qquad f'''(x) = (-1)(-2)(-3)\, x^{-4}$

$= -6x^{-4}$

Hence we claim $f^{(n)}(x) = \dfrac{(-1)^n \cdot n!}{x^{n+1}}$

M.I.

When $n = 1$, $f'(x) = -x^{-2}$ ✓

Assume $f^{(k)}(x) = \dfrac{(-1)^k \cdot k!}{x^{k+1}}$, then

$$f^{(k+1)}(x) = \frac{d}{dx} \frac{(-1)^k \cdot k!}{x^{k+1}}$$

$$= \frac{(-1)^{k+1} \cdot (k+1)!}{x^{k+2}}$$

Hence $f^{(n)}(x) = \dfrac{(-1)^n \cdot n!}{x^{n+1}}$

Q4. Let $y = \ln(x^2+2y^2)$.

Find $\frac{dy}{dx}$.

$$\frac{d}{dx} y = \frac{d}{dx} \ln(x^2+2y^2)$$

$$\frac{dy}{dx} = \frac{1}{x^2+2y^2} \cdot \frac{d}{dx}(x^2+2y^2)$$

$$\frac{d}{dx}(2y^2) = 4y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{x^2+2y^2} \cdot \left(2x + 4y\frac{dy}{dx}\right)$$

$$(x^2+2y^2)\frac{dy}{dx} = 2x + 4y\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x}{x^2+2y^2-4y}$$

Q5. Let $y = (x^2 - x)^{\ln(x)}$

Find $\frac{dy}{dx}$.

Note that $y = e^{\ln(x^2-x)\ln(x)}$

Sol: $\ln(y) = \ln(x^2-x) \cdot \ln(x)$

$$\frac{d}{dx}\ln(y) = \frac{d}{dx}\left(\ln(x^2-x) \cdot \ln(x)\right)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \left(\frac{d}{dx}\ln(x^2-x)\right) \cdot \ln(x) + \ln(x^2-x) \cdot \left(\frac{d}{dx}\ln(x)\right)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2x-1}{x^2-x} \cdot \ln(x) + \frac{1}{x} \cdot \ln(x^2-x)$$

Hence $\frac{dy}{dx} = y \cdot \left(\frac{2x-1}{x^2-x}\ln(x) + \frac{1}{x} \cdot \ln(x^2-x)\right)$

$$= (x^2-x)^{\ln(x)} \cdot \left(\frac{2x-1}{x^2-x}\ln(x) + \frac{1}{x} \cdot \ln(x^2-x)\right)$$