MATH 1010 Tutorial Oct. 8th (English III)

Presenter: WANG, Yizi
Email: yzwang@math.cuhk.edu.hk

Schedule:

5:30 – 6:05 Tutorial presentation
(5 Questions will be discussed)

6:05 – 6:15 Q & A
(Hosted by: YI, Tianhan)

Remark:

- You can visit yzwang.xyz or Blackboard to download the tutorial notes.

- MathGym (Faculty TA Q&A Centre) is now open. Please visit mathgym.math.cuhk.edu.hk for more information.

Q1: Let f be defined by

$$f(x) = \begin{cases} 7x^2 - 5m & \text{if } x \le 2 \\ 6x + 3m & \text{if } x > 2 \end{cases}$$

(a) Find (in terms of m)

(i) $\lim_{x \to 2^-} f(x)$ (ii) $\lim_{x \to 2^+} f(x)$

(b) Find the value of m so that

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

Sol: (a) (i) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2} 7x^2 - 5m = 28 - 5m$

(ii) $\lim_{x \to 2^+} f(x) = \lim_{x \to 2} 6x + 3m = 12 + 3m$

(b) $12 + 3m = 28 - 5m$

$8m = 16$

$m = 2$

Q2: Evaluate the limit

$$\lim_{x \to \frac{3}{4}} \frac{4x^2 - 3x}{|4x - 3|} \sim \frac{0}{0}$$

get rid of the absolute value

Observation:

$x > \frac{3}{4} \Rightarrow |4x - 3| = 4x - 3$

$x < \frac{3}{4} \Rightarrow |4x - 3| = 3 - 4x$

Left hand side limit:

$= x(4x-3)$

$$\lim_{x \to \frac{3}{4}^-} \frac{4x^2 - 3x}{|4x - 3|} = \lim_{x \to \frac{3}{4}} \frac{4x^2 - 3x}{3 - 4x}$$

$$= \lim_{x \to \frac{3}{4}} -x$$

$$= -\frac{3}{4}$$

Right hand side:

$$\lim_{x \to \frac{3}{4}^+} \frac{4x^2 - 3x}{|4x - 3|} = \lim_{x \to \frac{3}{4}} \frac{4x^2 - 3x}{4x - 3} = \frac{3}{4}$$

Hence the limit doesn't exist.

Q3: Evaluate the following limits:

(1) $\lim\limits_{x\to 1} \dfrac{x^2-1}{x^2-4x+3}$ $\sim \dfrac{0}{0}$

$x^2-1 = (x-1)(x+1)$ $x^2-4x+3 = (x-1)(x-3)$

$$\lim_{x\to 1} \frac{x^2-1}{x^2-4x+3} = \lim_{x\to 1} \frac{(x-1)(x+1)}{(x-1)(x-3)} = \lim_{x\to 1} \frac{x+1}{x-3} = -1.$$

(2) $\lim\limits_{x\to -\infty} \dfrac{x^2-1}{x^2-4x+3}$

$$\lim_{x\to -\infty} \frac{x^2-1}{x^2-4x+3} = \lim_{x\to -\infty} \frac{1-\frac{1}{x^2}}{1-\frac{4}{x}+\frac{3}{x^2}} = 1$$

(3) $\lim\limits_{x\to 2} \dfrac{\sqrt{x^2-3}-1}{x-2}$ $\sim \dfrac{0}{0}$

$(a+b)(a-b) = a^2-b^2$

$$(\underbrace{\sqrt{x^2-3}}_{a}-\underbrace{1}_{b})(\underbrace{\sqrt{x^2-3}+1}_{a+b}) = \underbrace{x^2-3-1}_{a^2-b^2} = x^2-4 = (x+2)(x-2)$$

$$\Rightarrow \quad \frac{\sqrt{x^2-3}-1}{x-2} = \frac{x+2}{\sqrt{x^2-3}+1}$$

$$\lim_{x\to 2} \frac{\sqrt{x^2-3}-1}{x-2} = \lim_{x\to 2} \frac{x+2}{\sqrt{x^2-3}+1} = 2$$

Q4: Use the given graph of the function g to find the following limits:

(1) $\lim_{x \to 1^-} g(x)$

$= 1$

(2) $\lim_{x \to 1^+} g(x)$

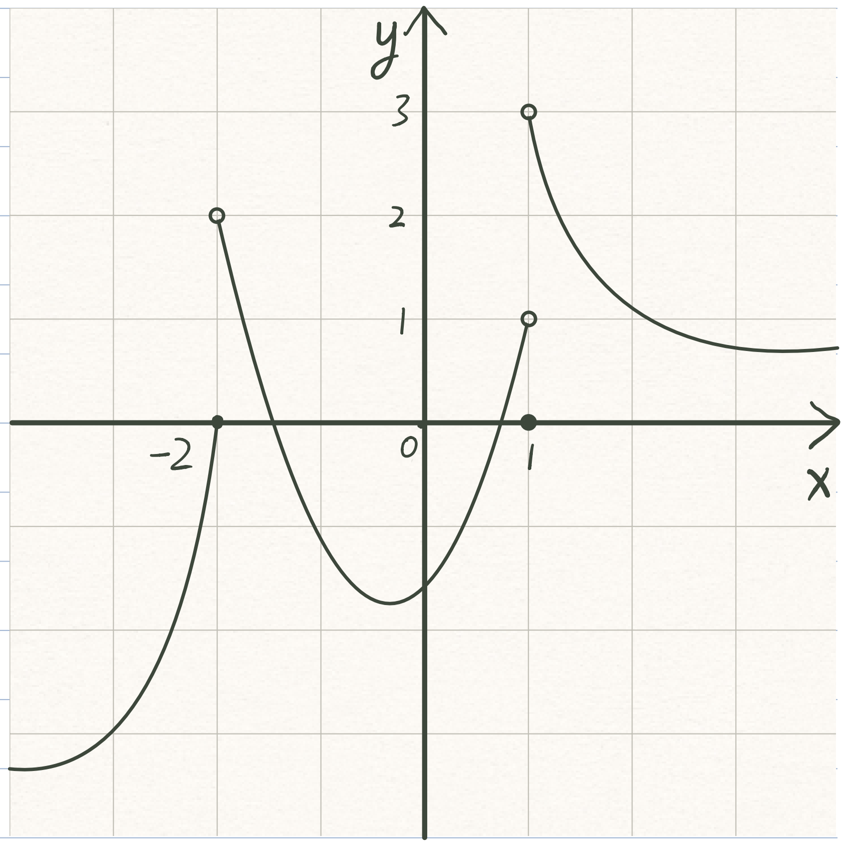
$= 3$

(3) $\lim_{x \to 1} g(x)$

D.N.E.

(4) $\lim_{x \to -2} g(x)$

D.N.E.

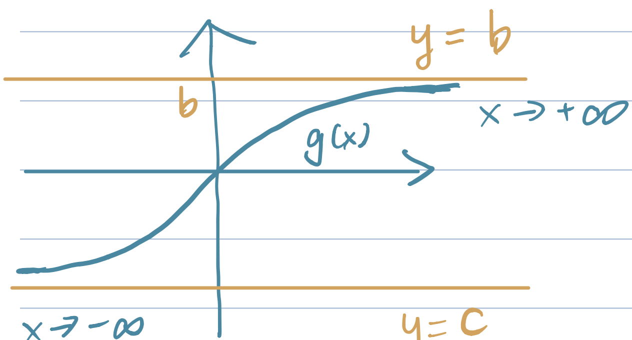


(5) $g(1)$

$= 0$

Q5: Let $f(x) = \dfrac{x}{\sqrt{x^2+1}}$.

Find the horizontal and vertical asymptotes of $f(x)$.

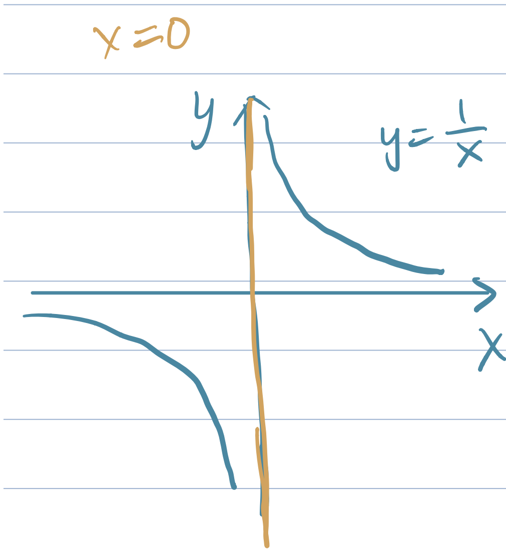


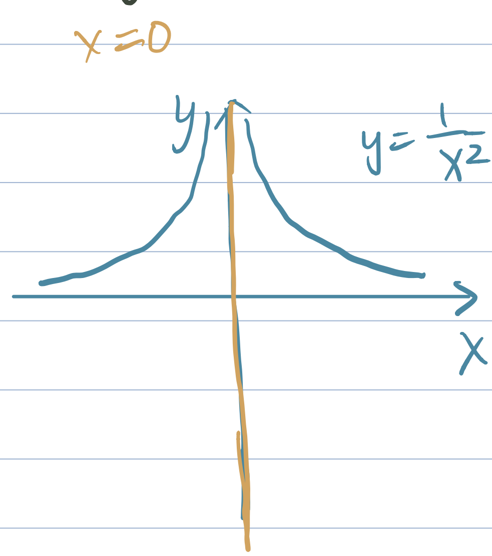
$$\begin{cases} \lim\limits_{x\to+\infty} g(x) = b \\ \lim\limits_{x\to-\infty} g(x) = c \end{cases}$$

$\Downarrow$

Horizontal asymptotes:

$$\begin{cases} y = b \\ y = c \end{cases}$$





$$\begin{cases} \lim\limits_{x\to x_0^-} g(x) = \pm\infty \\ \lim\limits_{x\to x_0^+} g(x) = \pm\infty \end{cases} \Rightarrow x = x_0 \text{ is a vertical asymptotes}$$

Sol: For $f(x) = \dfrac{x}{\sqrt{x^2+1}}$

① $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \dfrac{1}{\sqrt{1+\frac{1}{x^2}}} = 1$

$(x>0)$

$$\frac{x}{\sqrt{x^2+1}} = \frac{\sqrt{x^2}}{\sqrt{x^2+1}} = \sqrt{\frac{x^2}{x^2+1}} = \sqrt{\frac{1}{1+\frac{1}{x^2}}} = \frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

$\lim\limits_{x\to-\infty} f(x) = -1$ $\Rightarrow$ x negative

$\Rightarrow \sqrt{x^2} = -x$

$(x<0)$

$$\frac{x}{\sqrt{x^2+1}} = -\frac{\sqrt{x^2}}{\sqrt{x^2+1}} = -\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

Hence $y=1$, $y=-1$ are horizontal asymptotes

absolute value

② $|f(x)| = \dfrac{|x|}{\sqrt{x^2+1}} \le 1$. $\Rightarrow$ $-1 \le f(x) \le 1$

Hence there is no vertical asymptotes.